Real options and asymmetric volatility

In memory of Simon Benninga

Roi D. Taussig[1] and Sagi Akron[2]

ABSTRACT

This study offers a real options explanation for the asymmetric volatility phenomenon. The rationale is through the mechanism of real options exercise. Real call options add to the volatility of the underlying stock because they are equivalent to a leveraged buy of the stock. Real put options reduce the volatility of a stock because of their hedging effect. In a positive shock to returns, real call options are exercised, causing volatility to decrease. In a negative shock to returns, real put options are exercised, causing volatility to increase. We provide empirical evidence for our theory using Book-to-Market portfolios.

---

1. Introduction and literature review

The phenomenon of asymmetric volatility is well established in the finance literature. Asymmetric volatility means that aggregate market returns are negatively correlated with aggregate market volatility (e.g., French, Schwert, and Stambaugh (1987), Campbell and Hentschel (1992), and Duffee (1995)). There are two major explanations for the phenomenon and we provide a new explanation. The first explanation is leverage effect. When there is a negative (positive) shock to returns, firm's leverage increases (decreases), causing gross volatility to increase (decrease) (e.g., Black (1976) and Christie (1982). The second explanation is time-varying risk premium (a.k.a. volatility feedback, e.g., French, Schwert, and Stambaugh (1987), Bekaert and Wu (2000)). A positive shock to returns increases conditional volatility, but since volatility rises, there is an opposite force to reduce prices in order to account for higher risk premium. On the other hand, a negative shock to returns increases conditional volatility so the risk premium increases and pushes the prices even further downwards. In this case, the forces work in the same direction. The result is that after a negative shock to returns, conditional volatility increases significantly more than in a positive shock. The articles which support the volatility feedback explanation usually use the GARCH or TARCH methods (Bekaert and Wu (2000)). Avramov, Chordia, and Goyal (2006) add a trading based explanation for the daily asymmetric volatility phenomenon. They find that selling trading activity is connected to asymmetric volatility. After a daily decrease in stock prices, herding investors sell aggressively and push the stock prices further down, thus volatility increases. However, after a daily increase in stock prices, contrarian investors govern the selling activity, bring balance to stock prices, and thus decrease volatility.

The main contribution of our article is to provide a new explanation, based on the real options theory, for the asymmetric volatility phenomenon. Using the GARCH/TARCH methods, (Engle (1982), Bollerslev (1986), Glosten, Jaganathan, and Runkle (1993), Zakoian (1990))), it is evident that monthly conditional volatility for the U.S. market increases much more in a negative shock to returns than in a positive shock. We divide the market to 10 portfolios based on 10 deciles according to the NYSE break points of Book-to-Market. The asymmetric volatility is the highest in the top and bottom deciles. We hypothesize that the reason is exercise of real options. In the top Book-to-Market portfolio, investment is low and even negative, real put options are exercised, and conditional volatility increases significantly more in a negative shock to returns than in a positive shock. Real put options are used for hedging and reduce volatility. After their exercise, the volatility increases significantly more in a negative shock which strengthens the asymmetric volatility. In the bottom Book-to-Market portfolio, investment peaks, real call options are exercised, and conditional volatility increases relatively less in a positive shock to returns. Real call options increase conditional volatility and after their exercise in a positive shock to returns conditional volatility rises less than before the exercise. This strengthens the asymmetric volatility phenomenon. As we know from the binomial model (Cox, Ross, and Rubinstein (1979)), a call option can be duplicated by buying the underlying asset with a loan. The call option is actually a leveraged buy of stocks. This is why call options increase volatility. After the exercise of real call options in a positive shock to returns, volatility increases less than before the exercise.

A recent paper, *Real Options, Volatility and stock returns* (Grullon, Lyandres, and Zhdanov (2012)) explains the positive correlation (in Duffee (1995)) in individual stocks between returns and volatility. They also refer to the asymmetric volatility at the aggregate level. Though they analyze the phenomenon with real options like we do, they do not perform the analysis with a GARCH/TARCH method and they use gross volatility instead of conditional volatility. The causality in our analysis is different as well. In their analysis the volatility affects the real options while in our analysis the real options affect the volatility. Our analysis presents another explanation to the asymmetric volatility phenomenon present at the aggregate level. To some extent, this takes us back to the causality issue present in the leverage effect vs. volatility feedback. The returns affect the volatility or rather the volatility affects the return (Bekaert and Wu (2000)).

The remainder of the essay is as follows. Section 2 presents the data and analysis of asymmetric volatility at the aggregate level. Section 3 presents the data and analysis of 10 Book-to-Market portfolios. Section 4 presents the data and analysis of investment and exercise of real options. Section 5 concludes.

2. Asymmetric volatility at the aggregate level

It is well established in the financial literature that asymmetric volatility is present at the market aggregate level. We use the data from Kenneth French data library. The market return is the value-weighted return of all CRSP firms incorporated in the US which are listed on the NYSE/AMEX/NASDAQ and have a CRSP share code of 10 or 11 at the beginning of month t. Our sample is the market monthly returns from July of

year 1926 to December of year 2011. Table 1 presents the descriptive statistics of the

market returns.

Table 1
Descriptive statistics of the market returns.

Table 1 presents the descriptive statistics of the market return in percentage. MKT is the market returns. The market return is the value-weighted return of all CRSP firms incorporated in the US which are listed on the NYSE/AMEX/NASDAQ and have a CRSP share code of 10 or 11 at the beginning of month t. The sample is from July of year 1926 to December of year 2011. The division is for ranges of MKT, i.e., observations of MKT from (-40%)-(-20%) etc. It is clear that most monthly market returns are in the range of (-20%)-(20%).

MKT	Mean	Median	Max	Min.	Std. Dev.	Obs.
[-40, -20)	-23.55	-22.64	-20.59	-28.95	3.22	5
[-20, 0)	-3.76	-2.69	-0.010	-19.57	3.37	384
[0, 20)	3.71	3.17	16.66	0.00	2.78	632
[20, 40)	30.59	33.54	37.87	21.34	7.58	5
All	0.91	1.25	37.87	-28.95	5.43	1026

We run a GRACH/TARCH model (Engle (1982), Bollerslev (1986), Glosten, Jaganathan, and Runkle (1993), Zakoian (1990)) of the market returns according to the following specification:

$$MKT_t = C(1) + \varepsilon_t \tag{1}$$

$$\sigma_t^2 = C(2) + C(3) * \varepsilon_{t-1}^2 + C(4) * \varepsilon_{t-1}^2 * d_{t-1} + C(5) * \sigma_{t-1}^2 \tag{2}$$

Where $d_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, and $d_{t-1} = 0$ otherwise;

Table 2
TARCH results of asymmetric volatility for market returns

The table reports the coefficient of asymmetric volatility C(4) according to equations (1) and (2). Sample: 1926:07 2011:12. Included observations: 1026. Bollerslev-Wooldrige robust standard errors & covarian ce.

	Coefficient	Std. Error	z-Statistic	Prob.
C(1)	0.98	0.12	7.65	0.00
Variance Equation				
C(2)	0.96	0.35	2.71	0.00
C(3) ARCH(1)	0.06	0.03	2.05	0.04
C(4) (RESID<0)*ARCH(1)	0.11	0.04	2.69	0.00
C(5) GARCH(1)	0.84	0.03	23.21	0.00

As shown in table 2, C(4) is positive and significant, which means that there is an asymmetric volatility. In a negative shock to returns the conditional volatility rises significantly more than in a positive shock. In the next section we shall explain this phenomenon using the exercise of real options.

3. Asymmetric Volatility in Book-to-Market Portfolios

In order to explain the asymmetric volatility at the aggregate market level, we divide the market to 10 deciles according to Book-to-Market of firms. We use the 10 Book-to-Market portfolios from Kenneth French data library. Portfolios are formed on BE/ME at the end of June of year t according to NYSE BE/ME breakpoints. The stocks in the portfolios are from NYSE/AMEX/NASDAQ. BE of each stock is from fiscal year ending in t-1. ME is the price of a stock multiple by the number of shares outstanding in December of year t-1. The returns of each portfolio are the value-weighted monthly returns of the stocks in the portfolio. The sample is from July of 1926 to December of 2011. Table 3 presents the descriptive statistics of the returns of 10 Book-to-Market portfolios.

Table 3
Descriptive statistics of VW-returns of 10 BE/ME Portfolios

Portfolios are formed on BE/ME at the end of June of year t according to NYSE BE/ME breakpoints. The stocks in the portfolios are from NYSE/AMEX/NASDAQ. BE of each stock is from fiscal year ending in t-1. ME is the price of the stock multiple by the number of shares outstanding in December of year t-1. The returns of each portfolio are the value-weighted monthly returns of the stocks in the portfolio. The sample is from July of 1926 to December of 2011.

BM	1	2	3	4	5	6	7	8	9	10
MEAN	0.85	0.92	0.95	0.93	0.99	1.03	1.04	1.20	1.25	1.34
MEDIAN	1.02	1.17	1.11	1.24	1.19	1.22	1.31	1.26	1.47	1.27
MAX	38.77	34.82	31.23	57.13	46.36	58.35	61.64	71.76	64.31	102.29
MIN	-29.12	-26.83	-27.43	-24.41	-29.06	-34.16	-33.65	-31.41	-39.00	-45.46
Std.Dev.	5.74	5.52	5.35	6.08	5.67	6.22	6.67	6.99	7.59	9.40
OBS.	1026	1026	1026	1026	1026	1026	1026	1026	1026	1026

We run a GRACH/TARCH model (Engle (1982), Bollerslev (1986), Glosten, Jaganathan, and Runkle (1993), Zakoian (1990)) of each Book-to-Market portfolio according to the specification in equation (3) and (4):

$$BM_t = C(1) + \varepsilon_t \tag{3}$$

$$\sigma_t^2 = C(2) + C(3) * \varepsilon_{t-1}^2 + C(4) * \varepsilon_{t-1}^2 * d_{t-1} + C(5) * \sigma_{t-1}^2 \tag{4}$$

Where $d_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, and $d_{t-1} = 0$ otherwise;

Table 4
Results of GARCH/TARCH of Book-to-Market portfolios

The table presents the results of GARCH/TARCH model of 10 BE/ME portfolios. The dependent variable is the monthly returns on the Book-to-Market deciles from July of 1926 to December of 2011. There are 1026 observations for each portfolio. We use Bollerslev-Wooldrige robust standard errors and covariance. C (4) is the coefficient of asymmetric volatility.

BM	1	2	3	4	5	6	7	8	9	10
C(4)	0.18	0.06	0.06	0.13	0.11	0.09	0.15	0.15	0.14	0.16
Std.Dev.	0.04	0.05	0.04	0.04	0.04	0.03	0.04	0.04	0.03	0.04
P-Value	0.00	0.34	0.23	0.00	0.01	0.00	0.00	0.00	0.00	0.00

As shown in the results in table 4, the highest asymmetric volatility is in the highest and lowest Book-to-Market portfolios. In the highest Book-to-Market

portfolio, investment rock bottom, real put options are exercised, and conditional volatility increases substantially and significantly more in a negative shock to returns than in a positive shock. Real put options are hedging against negative shocks. In a negative shock to returns their value increases working as an opposite force to reduce conditional volatility. As investment shrinks substantially or even declines, real put options are exercised and as a result conditional volatility increases much more in a negative shock than in a positive shock. There is still the necessity to show that investment decreases substantially in the highest Book-to-Market portfolio. That will be shown in section 4.

In the lowest Book-to-Market portfolio, investment is the highest, real call options are exercised, and volatility increases *less* in a positive shock, causing asymmetric volatility to increase. Real call options are actually a leveraged buy of the firm which increases conditional volatility. When firms' investments peak, real call options are exercised, causing conditional volatility to increase relatively less after a positive shock to returns. Consistent with reality that the number of real call options dominates the number or real put options, only in the lowest Book-to-Market portfolio, investment peaks enough to exercise enough real call options and bring the asymmetric volatility to its peak. While the number of real put options is much lower in reality, as investment shrinks in higher BM portfolios, relatively many real put options are exercised, and asymmetric volatility rises quicker. In figures 1 and 2 we present the conditional volatility on shocks to portfolios' returns.
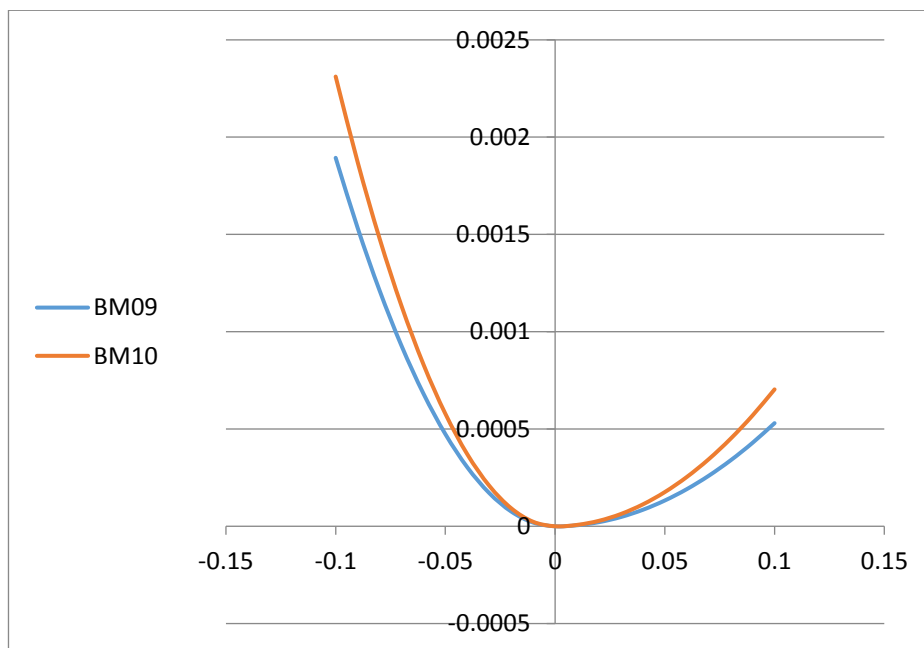
As shown in figure 1, in the highest Book-to-Market portfolio (BM10), when there is a negative shock to returns conditional variance increases more than in a

positive shock. It is also shown, that for portfolio BM10 the asymmetric volatility, i.e., the increase in conditional volatility for a negative shock to returns, is higher than for portfolio BM09. As mentioned, we maintain that this is caused by the exercise of more real put options. We will provide evidence in the next section that investment shrinks and even declines in portfolio BM10 relative to portfolios with lower Book-to-Market.

Figure 1

Conditional Variance impact curve for BM 09 & 10 portfolios' returns

The y axis is the conditional variance change for a change in the x-axis which is a shock to BM 09 & 10 portfolios' returns. The shocks to returns are from -0.1 to 0.1. BM10 is the highest Book-to-Market portfolio. BM09 is the second highest Book-to-Market portfolio. The curves are simulated according to the estimates of the TARCH models in table 4.


As shown in figure 2, in the lowest Book-to-Market portfolio (BM01), when there is a negative shock to returns conditional variance increases more than in a positive shock. It is also shown, that for portfolio BM01 the asymmetric volatility, i.e., the increase in conditional volatility for a negative shock to returns, is higher than for portfolio BM02. We maintain that this is caused by the exercise of more real call

options. Real call options increase conditional volatility. After the exercise of real call options in a positive shock to returns in portfolio BM01, conditional volatility increases *less* than in portfolio BM02. We will provide evidence in the next section that investment peaks in portfolio BM01 relative to portfolios with higher Book-to-Market. We show the magnitude of firms' investments in the Book-to-Market portfolios.

Figure 2

Conditional Variance impact curve for BM 01 & 02 portfolios

The y axis is the conditional variance change for a change in the x-axis which is a shock to BM 01 & 02 portfolios' returns. The shocks to returns are from -0.1 to 0.1. BM01 is the lowest Book-to-Market portfolio. BM02 is the second lowest Book-to-Market portfolio. The curves are simulated according to the estimates of the TARCH models in table 4.
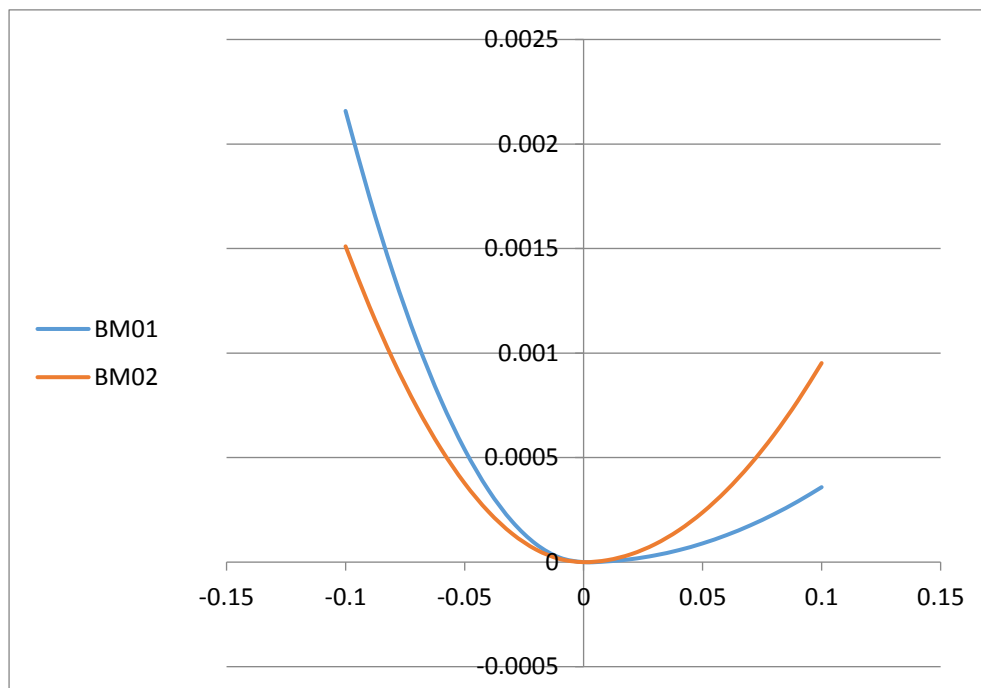
4. Investments and exercise of real options

The hypothesis of this chapter demands that in the highest Book-to-Market portfolio investment rock bottom causing the largest amount of exercise of real put options. The hypothesis also demands that in the lowest Book-to-Market portfolio investment peaks causing the largest amount of exercise of real call options. We use COMPUSTAT

10

data for the years 1964-2010 to examine the investment in the highest and lowest Book-to-Market deciles. As mentioned in Fama-French (1992), before 1963 COMPUSTAT data is biased. Book-to-Market break points are calculated according to method in section 3 (as in Fama-French (1992) and Kenneth French data library). We measure investment as the sum of total assets and R&D spending for all the firms in the Book-to-Market portfolio, all scaled by the sum of lagged total assets in the portfolio.

$$Investment_t = \frac{\sum_{i=1}^{N} AT_{it} - AT_{i(t-1)} + XRD_{it}}{\sum_{i=1}^{N} AT_{i(t-1)}}$$

(5)

The investment descriptive statistics in each Book-to-Market portfolio is presented in table 5. Consistent with our hypothesis, investment peaks in the lowest Book-to-Market portfolio, and rock bottom in the highest Book-to-Market portfolio. Investment peaks in the lowest Book-to-Market portfolio, which is consistent with the exercise of most real call options. Investment rock bottom and declines in the highest Book-to-Market portfolio, which is consistent with the exercise of most real put options.

Table 5
Investment descriptive statistics

The table presents the mean, median, and standard deviation of the investment for each Book-to-Market portfolio. Investment is measured as in equation (3.5) using COMPUSTAT annual data for the U.S. NYSE/AMEX/NASDAQ for the years 1964-2010

BM01	mean	0.15	median	0.15	Std.Dev.	0.06
BM02	mean	0.15	median	0.15	Std.Dev.	0.04
BM03	mean	0.14	median	0.14	Std.Dev.	0.05
BM04	mean	0.12	median	0.13	Std.Dev.	0.05
BM05	mean	0.11	median	0.11	Std.Dev.	0.04
BM06	mean	0.11	median	0.11	Std.Dev.	0.05
BM07	mean	0.09	median	0.10	Std.Dev.	0.15
BM08	mean	0.09	median	0.10	Std.Dev.	0.05
BM09	mean	0.07	median	0.07	Std.Dev.	0.05
BM10	mean	0.03	median	0.03	Std.Dev.	0.04

We also examine the difference in investment between the lowest and highest Book-to-Market portfolio and compute a t-statistic. The null is that the difference mean is equal to zero. The results are presented in table 6. The difference is highly significant different from zero. As shown, the null is significantly rejected even at the 0.01% level.

Table 6
Difference in investment between the lowest and highest Book-to-Market portfolios

Method	Value	Probability
t-statistic	13.65929	0.0000

5. Conclusions

The asymmetric volatility phenomenon is well established in the asset pricing literature. The main explanations to this phenomenon are the leverage effect and the volatility feedback effect. We employ the theory of real options in order to offer another explanation to this phenomenon which complements the volatility feedback effect. We divide the U.S. market on BE/ME (Book-to-Market) deciles to 10 portfolios.

The highest asymmetric volatility is present in the highest and lowest Book-to-Market portfolios.

In the highest Book-to-Market portfolio, investment rock bottom, most real put options are exercised and conditional volatility increases more in a negative shock to returns than in a positive shock. Real put options are used for hedging causing conditional volatility to reduce. In a negative shock to returns, real put options are exercised, causing the asymmetric volatility to increase. This effect is most dominant in the highest Book-to-Market portfolio.

In the lowest Book-to-Market portfolio, investment is the highest, most real call options are exercised in a positive shock to returns, and asymmetric volatility increases. Real call options increase conditional volatility because they are identical to a leverage buy of stocks. In the lowest Book-to-Market portfolio, in a positive shock to returns, these real call options are exercised, which causes the conditional volatility to increase less relative to before the exercise.

The evidences that we provide in the GARCH/TARCH models and the investment characteristics of the Book-to-Market portfolios support our hypothesis. More research is needed in order to explore and conclude which effect is most dominant in explaining the asymmetric volatility phenomenon and to what extent the effects are complementary or excluding each other.

REFERENCES

Avramov Doron, Chordia Tarun, and Goyal Amit, 2006, The Impact of Trades on Daily Volatility, *The Review of Financial Studies,* Vol. 19, No. 4, pp. 1241-1277.

Bekaert, Geert., and Wu, Guojun, 2000, Asymmetric Volatility and risk in Equity Markets, *The Review of Financial Studies,* spring 2000, Vol. 13, No. 1, pp. 1-42.

Bollerslev, Tim, 1986, Generalized Autoregressive Conditional Heteroskedasticity, *Journal of Econometrics 31, 307–327.*

Bollerslev Tim, Ray Y. Chou, and Kenneth F. Kroner, 1992, ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence*, Journal of Econometrics* 52, 5–59.

Bollerslev, Tim and Jeffrey M. Wooldridge, 1992, Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time Varying Covariances, *Econometric Reviews*, 11, 143–172.

Bollerslev, Tim, Robert F. Engle and Daniel B. Nelson, 1994, ARCH Models, in Chapter 49 of Handbook of Econometrics, Volume 4, North-Holland. , 539-580.

Bossaerts, P., and P. Hillion. 1999. Implementing Statistical Criteria to Select Return Forecasting Models: What Do We Learn? *Review of Financial Studies* 12:405–28.

Bossaerts, P., and P. Hillion. 1999. Implementing Statistical Criteria to Select Return Forecasting Models: What Do We Learn? *Review of Financial Studies* 12:405–28.

Christie, Andrew, 1982, The stochastic behavior of common stock variances: Value, leverage and interest rate effects, *Journal of Financial Economics* 10, 407–432.

Cox , John C. , Stephen A. Ross, and Mark Rubinstein. 1979. Option Pricing: A Simplified Approach. *Journal of Financial Economics* 7: 229-263.

Duffee, Gregory, 1995, Stock return and volatility. A firm-level analysis, *Journal of Financial Economics* 37, 399–420.

Engle, Robert F. 1982, Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation, *Econometrica*, 50, 987–1008.

Engle, Robert F. and Victor K. Ng, 1993, Measuring and Testing the Impact of News on Volatility, *Journal of Finance*, 48, 1022–1082.

French data library:

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Goyal, A., and I. Welch. 2003. Predicting the Equity Premium with Dividend Ratios. *Management Science* 49:639–54.

Grullon Gustavo., Lyandres Evgeni., Zhdanov Alexei., 2012, Real Options, Volatility, and stock returns, *Journal of Finance* 4, 1499-1537.

Zakoian, J.M., 1990, Threshold Heteroskedastic Models, manuscript, CREST, INSEE, Paris.